

METHODS OF CALCULATING CONTACT HEAT EXCHANGE.

I. HEAT CONDUCTION OF CONTIGUOUS IRREGULARITIES

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The physical laws of heat transfer through a zone of contact between solid bodies are analyzed in the paper. The influence of the height distribution of surface irregularities on the heat conduction of a contact between rough and wavy surfaces in a vacuum is studied in Part I. A system of three integral equations is formulated which allows one to determine the heat conduction α_s ($W/m^2 \cdot ^\circ K$) of contact spots in the general case as a function of the relative contact area, the contact pressure P (N/m^2), and the approach of the surfaces during compression. This system of equations admits of a simple analytical solution for elastic or plastic deformations of the irregularities if the height distribution of the irregularities is assigned in the form of a power function. For example, for the elastic deformations characteristic of the contact of wavy and rough, not coarsely worked surfaces the heat conduction of the contact is determined by the expression

$$\alpha_s = \frac{\lambda}{a} \left[\frac{Pa}{Ez} \right]^\omega,$$

where $\lambda = 2\lambda_1\lambda_2/(\lambda_1 + \lambda_2)$ is the effective coefficient of thermal conductivity of the contiguous bodies 1 and 2; $E = 2/[(1 - \nu_1)^2/E_1 + (1 - \nu_2)^2/E_2]$ is the effective elastic modulus; ν is the Poisson bracket; a is the maximum radius of the contact spots; $z = z_1 + z_2$ is the total arithmetic mean height of the irregularities; ω is the exponent, which takes a value of from 1/3 to 1 depending on the height distribution of the surface irregularities, i.e., on the cleanness and the means of treatment of the surfaces. It is interesting that whereas the mean radius of curvature r and the height of the irregularities vary by tens to hundreds of times as a function of the cleanness and the means of treatment of the surface, the maximum radius $a \approx \sqrt{rz}$ of the contact spots varies little and lies in the range of 10-30 μm for rough surfaces and 0.3-0.6 mm for wavy surfaces. In this case one can approximately assume that $1/3 < \omega < 0.6$ in the contact of wavy surfaces (most probable value $\omega = 0.4-0.5$), while in the contact of rough surfaces $\omega > 0.6$ (most probable value $\omega \approx 0.7-0.8$).

A comparison of the results of the calculation of the number, size, and heat conduction of contact spots with the experimental data of various authors shows that, on the whole, the calculation correctly reflects the main laws of heat exchange during the contact in a vacuum of both metal and ceramic solid bodies with rough or wavy surfaces.

Equations are also presented which enable one to estimate the influence of the discreteness of the contact spot itself and of the degree of oxidation of metal surfaces on the contact heat conduction. The results obtained indicate the important role of the height distribution of surface irregularities, the form of which controls the value of the exponent ω determining the dependence of the heat conduction of contact spots on the compression pressure and on the height of the irregularities.

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II. HEAT CONDUCTION OF GAS GAPS

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The influence of the interaction of a gas with contiguous surfaces on the heat exchange is studied theoretically. It is shown that the efficiency of the energy exchange between the gas and the walls plays a major role in the process of heat transfer through thin gas gaps. The thermal resistance $1/\alpha$ ($\text{deg}\cdot\text{m}^2/\text{W}$) of a contact gas gap of thickness δ can be treated as the sum of the resistance δ/λ due to the volume heat conduction and the thermal resistances $1/\alpha_{fm}$ of the gas-wall boundaries:

$$\frac{1}{\alpha} = \frac{\delta}{\lambda} + \frac{1}{\alpha_{fm}}, \quad (1)$$

where λ is the coefficient of thermal conductivity of the gas; $\alpha_{fm} = \xi P \sqrt{2R/\pi T}$ is the heat conduction of a gap due to free-molecule flow at a gas pressure P (N/m^2) and a temperature T ($^{\circ}\text{K}$); R is the gas constant ($\text{J}/\text{kg}\cdot\text{deg}$); ξ is the reduced accommodation coefficient.

The derivation of an equation for estimating the accommodation coefficient is given in the paper which correctly reflects the main relationships and agrees with the experimental data of various authors:

$$\xi = 1 - (1 - \xi_0) \exp(-U/kT). \quad (2)$$

Here $\xi_0 = \mu/2(1 + \mu)$ is the minimum value of the accommodation coefficient; μ is the ratio of the masses of atoms of the gas and of the wall; U is the threshold energy of the gas atoms (at a lower energy the atoms are adsorbed on the wall); T is the gas temperature. The accommodation coefficient is the smaller, the greater the difference between the masses of atoms of the gas and of the wall, the weaker their molecular interaction, and the higher the gas temperature.

The fact that the accommodation coefficient, in contrast to the coefficient of thermal conductivity of gases, is the larger, the greater the atomic weight of the gas leads to an interesting effect, in accordance with Eq. (1): The heat conduction of thin gas gaps proves to depend weakly on the nature of the gas filling the gap. This effect was discovered experimentally in the investigation of contact heat exchange in the fuel elements of nuclear reactors.

According to the calculations conducted, the mean geometrical thickness δ of a gas gap in Eq. (1) depends weakly on the contact pressure, and at low surface compression pressures it comprises $\delta = (0.5-1.0)z_0$, where z_0 is the maximum height of the irregularities, and at high pressures it decreases to $\delta \approx z$, where z is the arithmetic mean height of the irregularities.

By substituting into (1) the mean thickness of the gas gap and the free-molecule conduction for the accommodation coefficient (2), one can estimate the heat conduction of a gas in a contact gap as a function of the type of gas and walls, of the temperature and pressure of the gas, of the cleanness of the finish of the surfaces, and of their compression pressure.

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CONJUGATE PROBLEM OF HEAT EXCHANGE DURING THE NONSTEADY
LAMINAR FLOW OF A VISCOUS INCOMPRESSIBLE LIQUID IN
A SEMIINFINITE CHANNEL

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A converging finite-difference method is developed for solving the conjugate problem of heat exchange during the nonsteady, laminar, two-dimensional flow of a viscous incompressible liquid with constant thermophysical properties in the entrance section of a semiinfinite plane, annular, or cylindrical channel of constant cross section. The liberation of heat in the walls and the injection of coolant at the surfaces of heat exchange with the liquid are taken into account in the initial section of the channel. At the initial time a known disturbance of velocity and temperature is applied at the channel entrance, the heat sources in the walls are turned on, and the injection of coolant begins. The nonsteady process of flow and heat exchange which develops in the channel is studied. The initial conditions and the boundary conditions at the outer surfaces of the walls and the entrance faces are assumed to be assigned and consistent. The problem comes down to the calculation of the section of the channel which lengthens with time, whose right boundary withdraws from the entrance into the channel with a finite velocity, and which stays in a region of small temperature disturbances. The temperature boundary condition at it is assigned from the initial conditions, the transverse velocity of the liquid is assumed to equal zero, and the longitudinal velocity is determined on the basis of the equality of the liquid flow rate in the given section and its supply to the initial section of the channel. The problem is divided into an independent hydrodynamic problem and a thermal problem dependent on it. The hydrodynamic problem is described by the complete Navier-Stokes equations and is approximated implicitly with respect to velocity and explicitly with respect to pressure by a converging difference scheme of variable directions, suggested in [1]. The thermal problem is described by two-dimensional equations of heat conduction for the channel walls and by the energy equation for the liquid. The conditions of conjugation at the contact surface are set up in the form of boundary conditions of the fourth kind. With a known flow field the thermal problem is the problem of diffraction for equations of the parabolic type. For its approximation an economical difference scheme of fractional steps is set up, for which the stability is demonstrated and an estimate is obtained for the rate of convergence of the approximate solution to the exact solution in an energy norm. The coefficients of the difference equations, which depend on the velocity and its derivatives, are calculated at each step in time from the solution of the hydrodynamic problem. The algorithm is realized in the form of an ALGOL program for a BESM-6 computer and was tested on the example of a calculation of the steady heat exchange during airflow in a plane channel with thermally thin walls of copper. The calculation was carried out by the establishment method on a nonuniform spatial grid bunching near the channel entrance and the gas-wall contact surfaces. The calculated profiles of the temperature and Nusselt number coincide with those in [2] with an accuracy of 5 and 7%, respectively, which allows one to conclude that the method is workable.

LITERATURE CITED

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Longitudinal ribbon inserts in the form of a right helicoid are used to create twisted streams in pipes in order to intensify processes of heat and mass exchange, separation, and sedimentation. The problem analyzed below pertains to this field of engineering applications.

The steady laminar fluid flow in one of the two identical channels, into which the helicoid partition $\varphi = \sigma z$ divides the cylindrical pipe $r \leq 1$, is described by a system of equations (and boundary conditions) which can be converted to the form

$$L_i = \frac{1}{\text{Re}} \Delta_i (i = z, r, \varphi), \quad \frac{\partial}{\partial r} (rv_r) + \frac{\partial v_\psi}{\partial \psi} = 0; \quad (1)$$

$$v_i(r, 0) = v_i(r, \pi) = v_i(1, \psi) = p(0, \psi) = 0, \quad \langle v_z \rangle = 1, \quad (2)$$

where $z, r,$ and φ are the cylindrical coordinates; $\psi \equiv \varphi - \sigma z$; $v_i(r, \psi)$ and $p(r, \psi) - \omega z$ are the components of the velocity and pressure ($-\omega = \text{const} > 0$ is the longitudinal pressure drop); $v_\psi \equiv v_\varphi - \sigma r v_z$; $L_z = Dv_z - \left(\sigma \frac{\partial p}{\partial \psi} + \omega\right)$; $L_r = Dv_r - \frac{v_\psi^2}{r} + \frac{\partial p}{\partial r}$; $L_\varphi = Dv_\varphi + \frac{1}{r} \left(v_r v_\varphi + \frac{\partial p}{\partial \psi}\right)$; $D = v_r \frac{\partial}{\partial r} +$

$$\frac{v_\psi}{r} \frac{\partial}{\partial \psi}; \Delta_z = \Delta v_z; \Delta_{r,\psi} = \Delta v_{r,\psi} - \frac{1}{r^2} \left(\pm 2 \frac{\partial v_{\varphi,r}}{\partial \psi} + v_{r,\psi}\right); \Delta = \frac{1}{r} \frac{\partial}{\partial r} r \frac{\partial}{\partial r} + \left(\frac{1}{r^2} + \sigma^2\right) \frac{\partial^2}{\partial \psi^2}; \langle \dots \rangle = \frac{2}{\pi} \int_0^{\pi} \dots r dr \cdot$$

$\int_0^\pi d\psi \dots$ (here all the quantities are dimensionless: the linear quantities are normalized to the pipe radius α , the velocity to the longitudinal flow-rate velocity v_0 , and the pressure to the quantity ρv_0^2); σ is the ratio of the circumference of the pipe cross section to the pitch of the helicoid; $\text{Re} = \alpha v_0 / \nu$ is the "longitudinal" Reynolds number (the controlling criterion); ρ and ν are the density and kinematic viscosity of the fluid.

Integration of the system of four equations (1) with the conditions (2) determines the four unknown functions $v_i(r, \psi)$, $p(r, \psi)$, and the law of hydraulic resistance $\omega(\sigma, \text{Re})$ of the channel.

Let us note some of the conclusions following from the system of equations (1) in its general analysis.

The motion of the fluid in the channel cannot be fully helical ($v_\psi = v_r = 0$), and, in principle, it always, i.e., at any values of Re , includes secondary flows ($v_r \neq 0, v_\psi \neq 0$).

None of the unknown functions nor ω are one-term power functions of σ or Re . The latter two quantities cannot be reduced to one criterion, which their one-term combination would be.

In the particular case of $\sigma \ll 1$ with $\sigma \cdot \text{Re} = \text{idem}$ one can establish the following approximate dependences of the quantities on σ :

$$v_r \sim \sigma, \quad v_\varphi \sim \sigma (v_\psi \sim \sigma), \quad v_z = \text{inv}, \quad p \sim \sigma, \quad \omega \sim \sigma. \quad (3)$$

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VELOCITY OF TRANSLATIONAL MOTION OF MATERIAL IN A
ROTATING OVEN

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Equations for the velocity of the translational motion of material in a rotating oven are derived. The equations allow for the inclination of the axis of the oven to the horizontal and the inclination of the surface of the material to the axis of the oven. The latter is obtained in cases of variations of the diameter or physical properties of the material along the length of the oven and for support of the material by an annular baffle. The derivations of the equations are distinguished by extreme simplicity in comparison with those published in the literature. One variant of the equations makes it possible to directly allow for the effect of shelves on the motion of the material. In equations published earlier this effect is allowed for by means of an empirical factor. It is now clear that the separation of the factor is incorrect.

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HEAT CONDUCTION OF COMPOSITE STRUCTURES BASED ON
EPOXY COMPOUND AND METAL POWDERS

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The experimental investigation was carried out by the nonsteady method in the stage of an irregular thermal regime. The heat source (platinum wire) was placed in the plane separating two specimens, one of which was the test specimen while the second possessed known thermophysical properties: It was soft, assuring close contiguity to the test specimen and good contact with the heater upon compression. The measurement method was relative. To increase the accuracy the system of the balance method was used; i.e., the resistance of the bridge circuit was chosen so that the balance of the bridge circuit was retained during the entire time of pulsed heating, despite the heating of the platinum wires in the measurement and compensating cells. The circuit was balanced during the periodic application of electric heating pulses with a duration of 0.5 sec. The compensating circuit permitted control of the presence of contact thermal resistance at the boundary between the heater and the material in the course of the experiment. The error in measuring the heat conduction was no more than 5%.

The heat conduction of composites based on É-143 epoxy compound with a number of fillers, for which metal powders of zinc, nickel, and iron were used, was measured experimentally. The sizes of the metal particles were from 8 to 50 μm . The measurements were made at a temperature of 20°C; the dependence of the heat conduction of the composites on the volume concentration of the filler was studied. The experimental results show that only for zinc filling with particles of spherical shape does the heat conduction of the system coincide with the calculation for the heat conduction of mixtures with closed inclusions; for iron and nickel powders the agreement occurs in the region of low concentrations of filler, not exceeding 0.1. At higher concentrations the disagreement is considerable: The calculated data run considerably lower than the experimental data. The increase in the heat conduction of such systems containing components with high heat conduction can be connected with the formation of the mechanism of skeleton heat conduction. The formation of a skeleton from inclusions of nickel and iron is facilitated by the dendritic shape of their particles.

It is noted that for structures having inclusions with a high thermal conductivity the equation for the effective heat conduction can be obtained on the basis of the obvious concept of the "diffusion length of the temperature field."

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TEMPERATURE FIELD OF A THREE-LAYERED PLANE WALL WITH
BOUNDARY CONDITIONS OF THE SECOND KIND

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The temperature field of an infinite three-layered plate, heated by heat fluxes of constant power and with a uniform initial temperature distribution, is analyzed.

The solution of the stated problem, obtained with the help of a Laplace integral transform, consists of an asymptotic part and the sum of an infinite Fourier series. An analysis of the solution shows that at large Fourier numbers (Fo) the sum of the infinite series approaches zero and a quasisteady field of temperature gradients is established in the three-layered plate.

The value of the Fourier number, equal to $[Fo]$, beginning with which the temperature field at a given point N_1 is described with an assigned accuracy δ by the asymptotic part of the solution, is the time of onset of the quasisteady regime.

Since in practical calculations the sum of the infinite series is replaced by some partial sum n , we have $[Fo] = [Fo(\delta, N_1, n)]$.

An analysis of the dependence $[Fo] = [Fo(\delta, N_1, n)]$ shows that a calculation of the time of onset of the quasisteady thermal regime in a plate with allowance only for the first term of the series often leads to considerable errors in the determination of $[Fo]$; one observes a nonuniformity of the onset of the quasisteady regime over the thickness of the plate, due not only to the dependence of the sum of the series on the coordinate, but also to the parabolic character of the asymptotic part of the solution, with the coordinate N_1 of the minimum of the function $[Fo(\delta, N_1, n)]$ depending on the ratio of the heat fluxes at the plate boundaries and on the thermophysical characteristics and geometrical sizes of the individual layers of the plate.

The use of the solution obtained to calculate the temperature field of a three-layered plate with $Fo < [Fo]$ is connected with the necessity of calculating the sum of an infinite series. Therefore, one must preliminarily estimate the convergence of the Fourier series which enter into the solution.

For this purpose the sequences of partial sums $f_n(N_1, Fo)$ ($n = 1, 2, 3, \dots$) for different three-layered plates and heating regimes were studied on a computer.

From an analysis of the dependence $f_n(N_1, Fo)$ it follows that $f_{n \rightarrow \infty}(N_1, Fo = 0) = f_{n=(30-60)}(N_1, Fo = 0) \pm 10^{-4}$, with the convergence of the series improving with an increase in Fo , which allows one to determine the number of series terms sufficient for the calculation of the sum with any accuracy assigned in advance for an arbitrary Fo ; the absolute value of the sum of an infinite series is not always a monotonically decreasing function of time; in some cases the sum of the series is also nonmonotonic at $Fo > [Fo]$, which does not lead to violation of the laws of the quasisteady regime, however, when $\delta \geq 10^{-3}$.

In addition, the characteristic equation $\psi(\mu) = 0$ of the problem under consideration was investigated. An analysis of the function $\psi(\mu)$, which consists of the sum of four sines, made it possible to establish its period and the number of roots of $\psi(\mu)$ lying in an interval equal to the period, which considerably simplifies and simultaneously guarantees the search for all the roots of the characteristic equation.

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